Application of virtual pitch theory in music analysis

Llorenç Balsach

Abstract

In the course of this article a model of harmonic analysis is worked out based on certain properties of the auditory system, which I think will shed new light on the study of cadences and local harmonic resolutions. This model consists basically of extracting the two main fundamentals (roots) which are to be found in 93.3% of chords of less than 6 notes. To do this I apply the basic concepts of virtual pitch but taking into account only those harmonics which are in a prime position in the first seven, which in our thesis include the rest of the harmonics as far as the human auditory is concerned. With this model we found new information about the "internal" harmonic tension which is created by every single note and the tendencies of chords towards resolutions. I compare and discuss other models which apply the theory of virtual pitch in harmonic analysis.

Introduction

E. Terhardt's concept of "virtual pitch" is used in psycho-acoustics as a method of extracting pitch(es) from (harmonic) complex tone signals (see Terhardt, Stoll & Seewann (1982b, 1982c), Meddis & Hewitt (1991), van Immerseel & Martens (1992), Leman (1995)). The concept could be briefly described as the pitch which the auditory system perceives from a sound or group of sounds.

There have been some attempts to bring the acoustic concept of virtual pitch into musical theory. Terhardt himself (1982a) applied his theory to extract the root(s) of a chord. R. Parncutt (1988) saw that the results obtained from Terhardt's model did not fit in sufficiently with conventional musical theory; he worked out a revised version of the model, producing significantly different results from Terhardt's original.

Although theorists are in agreement about the concept of virtual pitch when applied to chords, its practical application to musical theory has been the subject of some discussion. As I have mentioned, the models proposed by Terhardt (1982a) and Parncutt (1988) differ considerably, especially in the results obtained from minor and diminished chords. They also differ considerably from Hindemith's notion of root (1937) and Riemann's classification of all chords into just three functional categories, (tonic, dominant and subdominant). When Terhardt, Stoll & Seeward's model (1982b, 1982c) for extracting virtual pitch(es) from complex tonal signals, was applied to chords played on the piano, the results from minor chords were not what would have been expected from Terhardt's model (1982a); results even differed depending on the inversion of the chord[1], something which reinforces Hindemith's theories but goes against the very essence of the concepts of root and virtual pitch. Finally, it will come as no surprise that all results obtained from all the above theories and experiments differ from the dogmatic definition of a root as the lowest of a series of notes ordered in thirds.

This article will discuss the models mentioned above and analyse the differences between the results obtained from each model. I will also propose a new model for functional

analysis of chords, based on the concept of virtual pitch and the "internal" tension which harmonics create in every single note.
Unlike earlier systems, the ease of application of this model allows it to be applied directly to the score without the need for a computer.
It is important to note that the model serves not only to extract the roots from chords or arpeggios, but also provides us with new information concerning the tension existing between chords and cadences. It also allows us to track the tonal centres of harmonically complex scores.

Virtual Pitch

The concept of virtual pitch applied to chords has its origins in the historic concepts of "basse fondamentale" (Rameau)[2], the "terzo suono" (Tartini)[3] and Riemann's harmonic-functional theory, concepts which in the course of history have unjustly come to lose their meaning as the basis of harmony: they eventually became mixed in with the rules of four voice harmony, bass continuo and the harmonic theory of grades. It was Terhardt himself (1982a) who was to bring back the relationship between the concept of virtual pitch and the musical concept of root, thereby giving the latter an acoustic dimension it had not previously had.
Terhardt's root(s) model (1982a) goes against traditional theory in that it has more than one root for a given chord: there are a series of roots, each one having its own importance as a representative of the chord. Terhardt also differs from traditional musical theory as regards the roots of minor chords.
The visual analogies of the concept of virtual pitch is shown in Fig.1 (Terhardt 1974). Virtual pitch in hearing would be similar to completing the outline of Fig. 1.
Terhardt's (1982a) and Parncutt's (1988) chord-root models are constructed in a similar way; in fact Parncutt's model is a revised version of Terhardt's models (1982a and 1982b). The basic process is to apply to chords the interval pattern of harmonics (up to the first 9

Fig. 1

Visual analogies of the concept of virtual pitch [Terhardt (1974)].
[from S. Coren, Psychol. Rev. 79, 359-367 (1972)].

partials). The notes and intervals are considered "pitch class" and "interval class", i.e. octave equivalence and equivalence between inversions is applied[4].
The procedure for Terhardt's model is as follows: we have to find all the possible "subharmonics" of each note in a chord. By "subharmonic" we mean a pitch which has an interval relationship lying within the natural harmonic pattern. That is, the subharmonics of a note are the pitches of which the note can be considered a "harmonic". Each

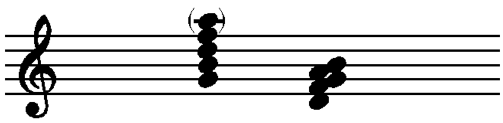
subharmonic is a theoretical virtual pitch. The pitch which is most often a subharmonic of the notes of a chord is most likely to be the root of that chord. Such are the basic premises of Terhardt's model (1982a).
The interval pattern of the harmonics is equivalent to the interval structure determined by the chord CGEBbD (the first 9 harmonics beginning with C as root and discounting any repeated notes).

For example, taking the chord ACE:
- the subharmonics of A are A (pattern C:C), D (pattern C:G), F (pattern C:E), B (pattern C:Bb) and G (pattern C:D).
- C has C, F, Ab, D and Bb as subharmonics.
- E has E, A, C, F# and D as subharmonics.

In this way D features three times and A, C, and F twice, which is to say that all three notes of the A minor chord are harmonics of D and that two notes of the chord are harmonics of A, C and F. According to this model D would have the highest "score" (3 appearances) to qualify as the root of the chord, followed by A (2), C (2) and F (2).
Parncutt (1988) points out that this model (Terhardt, 1982a) does not explain minor chords satisfactorily from the point of view of traditional theory, which gives the pitch of A and not D as the root of the above chord. Parncutt, clearly influenced by traditional theory, proposes a revised version of Terhardt's model.
Basing his argument on the similar nature of the interval classes occurring between the 3rd. and 5th. and the 3rd. and 7th. partials Parncutt maintains that the auditory system hears a kind of subharmonic a third below the fundamental[5], and therefore adds the minor third as support for a root. The rest of Parncutt's model is similar to Terhardt's, but he establishes a speculative series of "weights" for each harmonic.
Parncutt revised Terhardt's model because he thought that the results so obtained did not tally with musical theory. Possibly he was right, but what exactly does musical theory say concerning the root of the minor chord?

Conventional musical theory often fails to make an adequate distinction between two very different concepts —(con)sonance of a chord and a chord's function as a generator of harmonic tension. (Con)Sonance refers to the greater or lesser sensation of smoothness or roughness we experience on hearing the chord, while the function of a chord refers to the harmonic tension which that chord creates.

Fig. 2



For example, the two chords shown in fig. 2 share the same harmonic function as they both create a tonal vector towards C. Their degree of sonance, however, is totally different.

In this example the root is the same whether we define it as the bass of the "root state" (ordering the notes in thirds), or as the representative of the function of the chord. But the bass of a chord ordered in thirds is not always the same as the root defined representative of the function of the chord. This is true in the case of minor chords.
The chord ACE probably has its greatest consonance when A is the bass note. In conventional theory A is therefore the root of the chord. But students of harmony soon learn that they cannot use this chord as the dominant of D and so A, which is the dominant of D, cannot represent the function of the chord. For functional harmony A is not the root of the chord. This confusion concerning the root of the minor chord is due to the fact that we are talking about two different ideas —on one hand the (con)sonance of a chord and on the other its function.

In the 18th. century Rameau had already defined the chord ACEG as having a double function with A or C as the "basse fondamentale", depending on the harmonic progression or the inversion (A as a "sixte ajoutée").
When Riemann classified the functions of all chords into just three categories, (subdominant, tonic and dominant) he made the D minor chord in the key of C major into a subdominant (he anotated it Sp) (root F); the chord of E minor became a dominant (Dp) (root G); the A minor chord became a tonic (Tp) (root C), and the diminished chord BDF was a dominant with G as its virtual root. Riemann was in agreement with Tartini, many years before this concept was defined acoustically by Terhardt.
All this tends to indicate that from the point of view of functional harmony, the root of chord ACE is C.

In this way musical theory seems to give A as bass to underpin the most consonant chords and C to represent the function of the A minor chord.

Yet another interpretation of the root of a minor chord is that maintained by the "dualist" or "polarist" theory (see Forte, Vogel, Levarie...): here the minor chord is an inverse reflection of the major chord, and therefore the chord ACE has the pitch E as its theoretical root.
As we have seen, Terhardt and Parncutt also add the virtual roots of D and F.

As we can see, there is a wide variety of opinions concerning the "root" of a minor chord. These differing opinions also occur when considering other types of chord. The problem is clearly caused by the lack of a clear definition of the term "root", which is used for four different things —the bass as the lowest of a series of notes ordered in thirds, the bass note of the most consonant chord, the pitch which represents the function of a chord and the pitch which represents a chord acoustically.

Having defined the concepts of virtual pitch and described the differing opinions concerning the concept of "root", in the following pages of this article I will propose a model of local harmonic analysis which takes into account both the concept of virtual pitch and that of harmonic tension.

Prime harmonics

Since Aristoxenos, and for the majority of later theorists including Zarlino and Rameau, harmonic theory was based on the arithmetic ratios of the numbers 1, 2, 3, 4, 5, and 6 (known as the "senario")[6]. Some theorists, such as Leibnitz (see Luppi, 1989), Tartini (1754 and 1767) Euler, Kirnberger and Vogel (see Vogel, 1993) add the number 7 to the "senario".

Since Rameau (1750) it has been known that harmonically establishing the arithmetic relations of the first 6 or 7 natural numbers is equivalent to considering the relations for the human auditory system of the first 6 or 7 partials of a (harmonic) tone. Those in favour of including the number 7 consider the minor seventh as harmonic ratio 7/4, while those in favour of the "senario" consider the minor seventh as the ratio of two fifths/fourths (C:F:Bb) (4/3 x 4/3). Riemann himself, while being in favour of the "senario" for tonal relations, accepted that the interval of a minor seventh was a direct gift of nature (Elementar-Musiklehre, Hamburg, 1883.)[7].

In psycho-acoustics it is considered that there is no reason to "cut off" the harmonic series for a given position, especially for such a low number as 6 or 7. If this is done it is purely for practical reasons. "Cutting off" is carried out in higher positions, from 8 (Plomb 1964) to 15 (Leman 1995). In many cases the harmonic series is not cut off when constructing the algorithms: each partial is given a weight inversely proportional to its position in the acoustic spectrum.

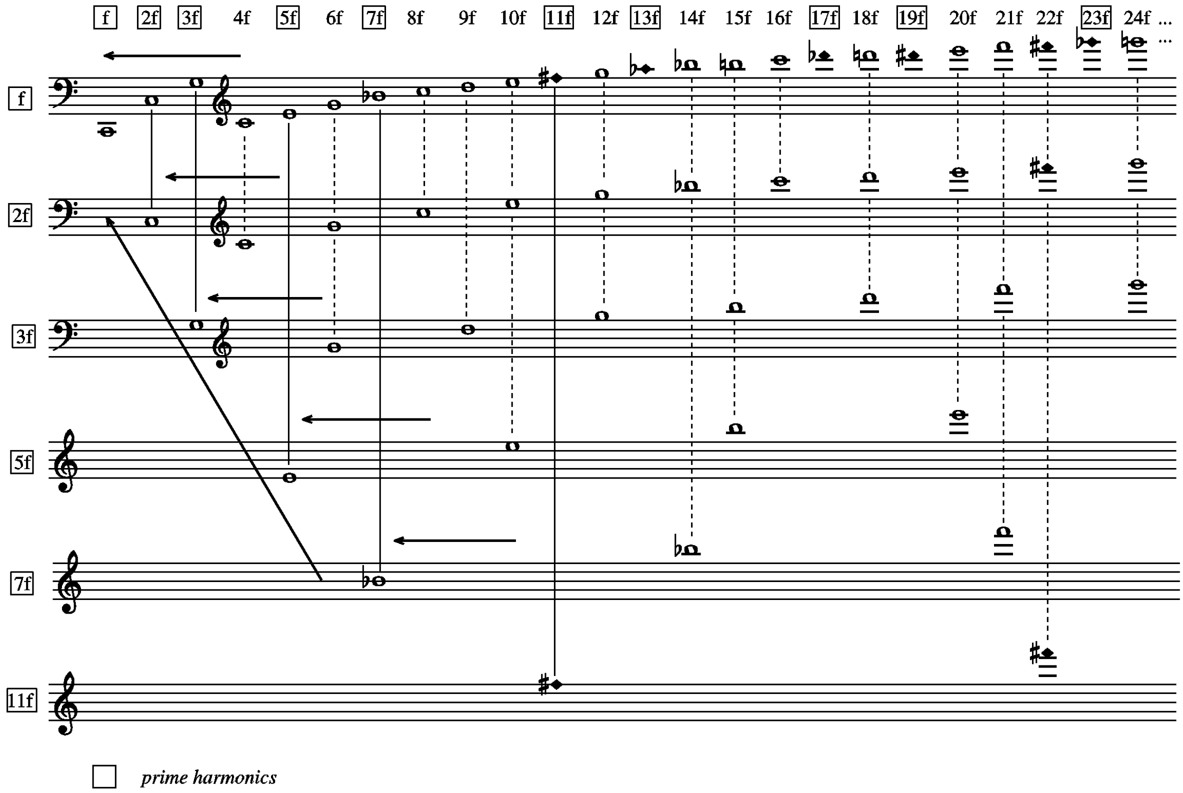
However, the results obtained from calculating the possible virtual pitches can vary considerably depending on the number of harmonics which are taken into account, or on the weight given to each harmonic. The point at which we "cut off" the harmonics, or their "weight", is therefore of considerable importance.

I believe that we can take into account a sufficiently high number of harmonics (up to the 24th. for example)[8], although it is the first seven which are really important for the auditory system.

The reason is that those harmonics which are multiples of a prime harmonic are less important for the auditory system than those which are in a prime position. In the same way as a fundamental is the psycho-acoustical synthesis of all its harmonics, prime harmonics are the synthesis of the rest of the multiple harmonics ("harmonics of the prime harmonics"). We could take into account two levels of perception (see fig. 3):
1) The fundamental as perception of all the (prime) harmonics, and
2) the prime harmonics as perception of all the multiple harmonics.

The weight of each prime harmonic depends on how many of its multiple harmonics the auditory system can hear and distinguish.

Fig. 3 Prime numbers as generators of harmonic perception



The results shown in fig. 3 reflect the history of harmonic theory in regarding the first five to seven harmonics to be those that are functionally perceived by the auditory system.

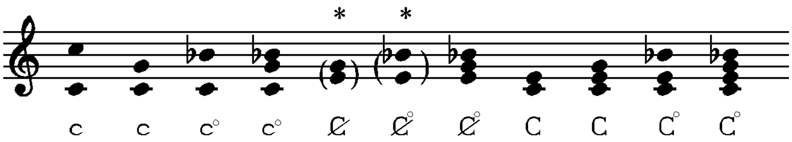
Convergent chords as virtual pitch patterns

Our model will therefore take into account only the first 7 harmonics (prime harmonics 2,3,5 and 7), unlike Terhardt and Parncutt who consider the first 9. The pattern structure is therefore the chord CGEBb, which I will call the complete convergent chord (as with Terhardt and Parncutt, considered as a chord class, i.e. the notes can be in any order).

My approach to virtual pitch differs from the system of matching subharmonics used in Terhardt and Parncutt's models. In fact our model (Balsach, 1994) was worked out some years ago when I was still unaware of the models of these two analysts. For this reason the methodology used is somewhat different. I think more in terms of interval patterns than of subharmonics, although we will see that if we consider only the first seven harmonics, the resulting virtual pitches are similar.

Our vision of virtual pitch could be summed up as follows: any part of the pattern of the convergent chord CGEBb has C as its virtual pitch (any combination of the notes in the chord tends towards C). This is due to the acoustic properties which can be observed when we filter out certain partials of a (harmonic) complex tone. Here the impression of fundamental does not change, even if it is that very fundamental which is eliminated from the chord. For example chords EG, EGBb, BbCG, EC, BbC, EBb, etc. all tend towards C (have C as their fundamental). All these chords, formed by parts of the structure of the 7 first harmonics, I have called "convergent chords". (fig. 4)

Fig. 4 Convergent chords



* *In pure tuning*

If we listen to these convergent chords as pure tones (naturally tuned sinusoidal waves), we will indeed see that in each case the pitch C is perfectly audible.[9]

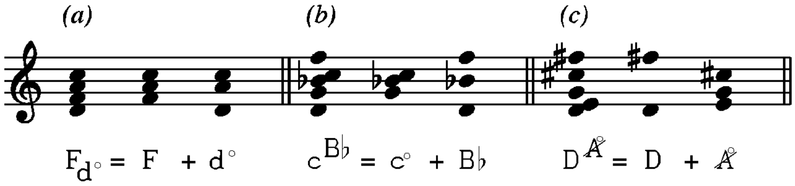
When we adjust the interval substructures of the complete convergent chord for equal temperament, we come up against two problematic interval classes.

The first of these is the equivalence of intervals E:G and G:Bb. As my aim is to simplify our model as much as possible, we will say that for practical purposes the first of these intervals inhibits the second. There are two reasons for this: first, I consider harmonic E to be more poIrful than harmonic Bb, and secondly the natural interval E:G is closer to the equal temperament interval of a minor third than is the natural interval G:Bb. Even though I discount this interval for practical purposes, we must always remember that interval G:Bb has a weak virtual pitch on C (or what comes to the same, interval E:G has a weak virtual pitch on A).

The second difficult case is the equivalence in equal temperament of the inversion of the interval E:Bb. This causes the interval to have two virtual pitches —C, when the auditory system hears E:Bb, or F#, when the auditory system hears A#:E. Our model will only take into account two roots when the harmonic notation of a chord is ambiguous; otherwise we will take it that the auditory system perceives only one root. For example in chord EBbF the auditory system cannot perceive Bb as A#, as E and F support Bb, not A#. The two main roots are therefore C(E:Bb) and Bb(Bb:F), not F#.

With this approach to virtual pitch, i.e. according to convergent structures, we find on analysis of all existing chords of up to five notes that 93.3% of them can be separated into two convergent chords (Balsach, 1994). This amounts to saying that the majority of chords have two significant roots (fundamentals) (see fig. 5). We will see that this simplification of the concept of virtual pitch is a great help in harmonic analysis. The separation of any kind of chord into two convergent chords may seem at first sight a difficult business, but it will become easier when we see that of all the intervals which make up the pattern structure of the convergent chord, only two have functional importance —the major third (C:E) and the diminished fifth (E:Bb).

Fig. 5 Separation of chords into convergent chords



I will henceforward use the term "fundamental" instead of "root" when referring to the convergent components of any chord.

Internal tension of a single note

How is it that if we play a single note on any instrument, some of the subsequent notes sound as if they tend towards "resolution" —they produce a feeling of relaxation which other notes do not?

Why does playing F after C give an impression of resolution, even when these notes are isolated from all tonal memory? Why do we have this impression of resolution or relaxation when we play E after F?

The answer to these questions turns out to be surprisingly simple once we have brought down the harmonic spectrum of a note to the convergent chord CEGBb.

It is well known that since the early days of musical theory the two intervals considered to be most consonant have been the octave and the fifth. A quick glance at fig. 3 shows why. The consonance is due to the great number of harmonics these two notes have in common. The octave has so many harmonics in common with the fundamental that the auditory system considers it to be "the same note" but with a "special timbre". This is due to the acoustic effect mentioned earlier, which can remove harmonics from the spectrum without causing any significant variation in perceptiom of the fundamental pitch. In the case of the fifth this "identification" occurs at a different level of perception. An interval very close to the octave or the fifth will be perceived by the auditory system as a "deformation" of the pure consonance: as a result it causes tension at the interval in question.

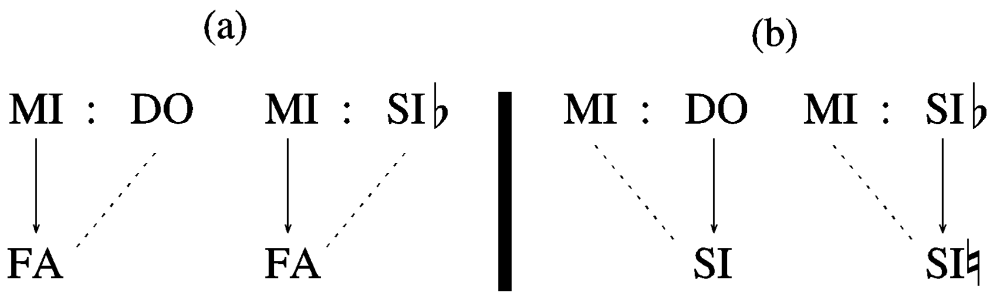
If we look at the chord CEGBb there are two "quasi-fifths" or "false fifths" —interval classes E:C and E:Bb— which differ from the perfect fifth by just a "semitone". The pitches are still closer to the perfect fifth if we are working with equal temperament.

The note E is the main actor in this internal conflict. On one hand it is a natural tone in that it is the 5th. harmonic of the fundamental C, whilst on the other it is the cause of a conflict which amongst other harmonics creates the false fifths E:C and E:Bb.

This contradiction creates a degree of instability in the chord CEGBb and consequently a degree of internal tension at pitch C.

"False fifth" E:C can resolve onto the perfect fifths F:C or E:B, i.e. the notes of F or B are "resolutions" of this internal conflict. But it is surprising to see that these two notes (F and B) are resolutions of the other internal tension formed by the false fifth E:Bb. This can resolve onto the perfect fifths Bb:F and E:B (among other two, always taking the fifths as interval classes) (see fig.6) I.e. tone C, which is the final result of the structure CEGBb, resolves onto F or B.

Fig. 6



Resolution of pitch C onto F has the added value that at the same time the auditory system recognizes C as the 3rd. harmonic of F. This is the usual explanation for the tendency C displays towards F, but as we have seen, it is not the only explanation. It is well known that a greater sense of resolution is perceived when we hear the fifth rather than the octave below a note. This proves that recognition of notes as close harmonics is not the main explanation of resolution, as the octave is the 2nd. partial and is closer to the fundamental than the fifth. The theory of harmonic proximity would have the octave as possessing more resolving power than the fifth. The fact that the sense of resolution is greater with the fifth than with the octave is due to this internal tension of a single tone.

To sum up briefly, there is internal tension in every note. This tension has two possible resolutions —the main resolution on the fifth below the note and a secondary resolution on the minor second below (taking the intervals as "interval classes"). If we apply this to virtual pitches, a fundamental (as virtual pitch) will have its main resolution towards a fundamental which is a fifth below (or a fourth above), and its secondary resolution towards a minor second below (or major seventh above). In this way C (as single tone or fundamental) has two tendencies towards resolution —towards F and B. Note that these two resolutions are at opposite poles in the circle of fifths.

Surely this double tendency towards resolution is the underlying "raison d'être" of all the rules governing cadences in harmony and tonality in musical history.

Up to the 16th. century these two cadences were used together with those cadences that were understood as "resolutions" of dissonances between the voices. As from the 16th. and 17th. centuries cadence formulae based on a jump of a fifth began to displace other formulae, although Phrygian formulae (a jump downwards of a minor second from the

main virtual pitch) were still fairly common, especially in Monteverdi. As the rules of tonality gradually became established in the course of the 18th. century, the "authentic" cadence began to become the absolute norm for cadential resolutions. The reason for the disappearance of Phrygian formulae was that in the major and minor scales of classical tonality, fundamental movements (taken as virtual pitch(es)) of a minor second below could not be carried out without chromaticisms. (An exception was iv-V or VI-V in the minor mode with leading tone, a link that was considered cadential). The cadential power of this joint movement is sufficiently great for the formula also to be used in classical tonality, hidden within other cadential formulae such as Neapolitan cadences and the use of the German augmented sixth. In classical Greece the descending Dorian scale (equivalent to the Phrygian ecclesiastic mode) was considered to be the "authentic Hellenic mode": this was formed by two identical tetrachords, EDCB and AGFE, with resolutions on movements C-B and F-E. Plato took this mode as a model of order for the Republic[10]. "Resolution" onto a tonic C (predicted by tonal diatonic system) would have been unthinkable for the Greeks.
Use of Phrygian cadences in folk music is well known, especially in my country (Spain).

The functional fundamentals of a chord

In the foregoing pages we have seen that virtual pitches are decided by the structure of the first seven prime harmonics, represented by the interval pattern of the chord CEGBb , that is interval classes M3, P5 and m7, determining a "real fundamental C" and the interval classes m3 and d5 which determine a "virtual fundamental C" (see fig. 4). As the majority of chords of 5 notes or less can be separated into two convergent chords, chords of up to 5 notes have two main (real or virtual) fundamentals. The functional weight of the two or more fundamentals of a chord is established by the tension created by the first 7 partials of any harmonic tone.

As we have seen, tension of a convergent chord (with a real or virtual fundamental C) is mainly caused by the note E. Of the four main prime harmonics (nums. 2, 3, 5 and 7), num. 5 (interval class M3) is therefore of the utmost importance. I have used the term "functional fundamental" to designate the (real or virtual) fundamental in a chord containing its major third, as it will tend to resolve towards the fifth or minor second below.

So that all these properties of the chords are included in our model of harmonic analysis, I will use simple terminology for each type of fundamental. I will use capitals for the fundamentals of convergent chords which include their major third, and small letters for the fundamentals of convergent chords which do not have this interval. I will also distinguish between those convergent chords which include a Bb (the 7th. partial) and those which do not, and between those which have a real or virtual fundamental. All of this is illustrated in fig. 4. I make no distinction between chords with or without the octave or the fifth (of the fundamental): as can be proved by traditional theory, these notes do not affect establishment of the functional properties of chords, being notes which are closely related to the fundamental and to some degree identified with it.
Musical theory takes this for granted as far as the octave is concerned, and there are no consequences if this note is omitted. But the significance of the fifth as a powerful,

consonant interval is often CONFUSED with its significance as FUNCTIONAL SUPPORT for a root. The intervals of an octave and a fifth are the most consonant and closest to the fundamental, and they therefore have the LEAST FUNCTIONAL importance when they appear in a chord!

I will now apply everything that has been explained above to the minor chord ACE. This chord has the best separation into convergent chords AE and CE, thereby having two main fundamentals on C and a. C is the "functional fundamental", as it has an interval of a major third C:E. This chord will tend to resolve towards fundamentals F and B. This is a simplified result from our model, satisfactory for analysing with pen and paper. If we study the chord in more detail we will find a weaker virtual pitch determined by the interval of a minor third (convergent chord A:C taken as the 5th. and 3rd. harmonics), giving us a virtual fundamental F (pattern F:A:C); there is yet another, even weaker, virtual pitch determined by the same interval (convergent chord A:C taken as the 3rd. and 7th. harmonics, giving a virtual fundamental D (pattern D:F#:A:C).
If we use Terhardt's model but limit it to the first 7 harmonics —the original model went up to first 9—, roots D, F, A and C will also have the highest "score" (all 4 pitches with 2 "marks").
If we use Parncutt's model with only the first 7 harmonics, we will also find that pitch classes D, F, A and C have the highest "score", but in this case the scores are A (1.60), C (1.33), F (0.83) and D (0.75).

Next our model will be applied to the chord CEbGbBb, which Parncutt also analyses in his article.
This chord can be separated into convergent chords CEbGb and GbBb, giving fundamentals Ab and Gb (patterns Ab:C:Eb:Gb and Gb:Bb).
Both fundamentals "possess" their major third. They are therefore functional fundamentals and are notated in capitals. Ab is a virtual fundamental and Gb a real fundamental (it is present in the chord). Both of these fundamentals are equally important. This chord will tend to resolve towards chords based on fundamentals Db, Cb, (G) and F (see next section).

Terhardt's model (7 harmonics) gives Ab as the main root and Gb, C, D, Eb, F and Cb as secondary roots. Parncutt's model has Eb as the main root and C, Gb, Ab and Bb as secondary roots.

I believe that if a chord can be separated into two convergent chords, we need no more than the fundamentals of these convergent chords to represent the chord in question, as they include all the notes of the chord and we do not need to look for weaker virtual pitches which would make practical harmonic analysis more difficult.

Application of the theory of convergent chords to harmonic analysis

A convergent chord is represented by a single true fundamental. We have seen that these fundamentals "resolve" onto other convergent chords whose fundamentals are a fifth or a minor second below, see fig. 7 (local relaxation, independent of a tonal field)[11].

Fig. 7 Homotonic relaxation between fundamentals

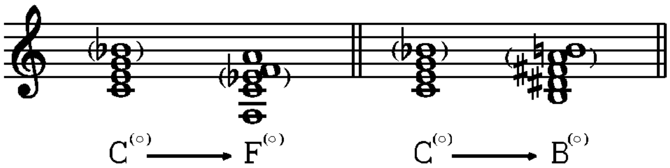


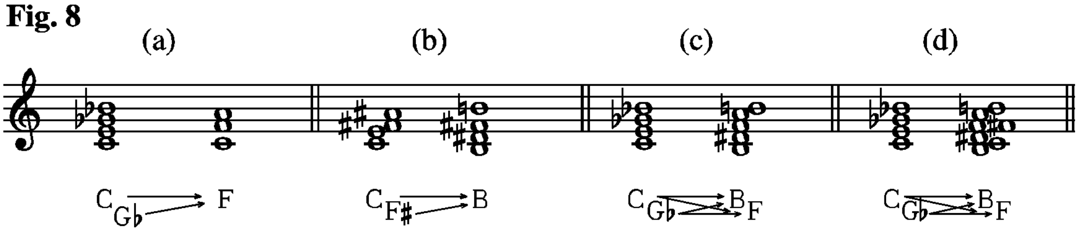
Fig. 6 shows that resolution of the false fifths C:E and E:Bb creates interior movements of a rising or falling "semitone". These two "melodic" resolutions are closely related to the two "harmonic" resolutions mentioned above, but they must be considered to be of a different type and deduced from the "harmonic resolution". A rising semitone resolves melodically between notes but this resolution never applies to fundamentals. We could say that C resolves onto B "harmonically" and B resolves onto C "melodically" as a result of the auditory system associating these movements with the interior movements of the harmonics to correct the internal "false fifths". From the point of view of local harmonic analysis the succession of fundamentals C-B relaxes the harmonic tension, while the succession B-C increases it; in the same way succession C-F reduces local tension while succession F-C increases it.[12]

When a chord is formed by two convergent chords there are two fundamentals: if they are functional fundamentals (i.e. they include the 5th. harmonic —M3—) they will both tend to resolve towards a fundamental a fifth or a minor second below. Resolution of compound chords can be considered a result of the resolutions of their fundamentals.

Resolution of one of the fundamentals is not always satisfactory for the chord as a whole, as tension can be created with the other fundamental. For example, of the two resolutions *(Db and G)* of the fundamental Ab in the foregoing chord (CEbGbBb), the resolution on G creates tension with the other fundamental Gb. However, the resolution on Db is a good one as Db is a harmonic of Gb.

If a fundamental also reduces the tension of the two foregoing fundamentals, its cadential direction is clear, even if the second chord is more dissonant that the first. In fig. 8a both C and Gb resolve onto F, so we have a clear harmonic resolution, independent of the sonance resolution. Fig. 8b shows the same situation, as both C and Gb (F#) resolve onto B. In fig. 8c chord BF clearly reduces harmonic tension as in any combination B and F are always resolutions of fundamentals C and Gb, even though the chord is of equal or greater dissonance than the previous one. Even in fig. 8d we have harmonic resolution against strong sonance tension.

Fig. 8

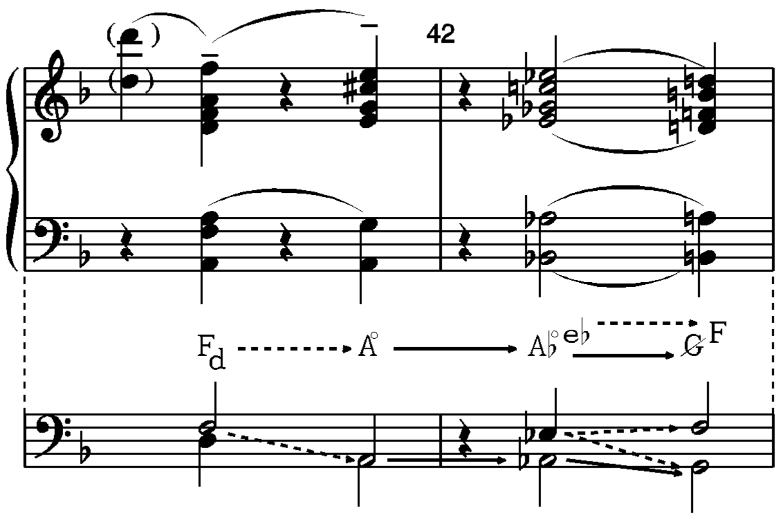


The harmonic resolution in fig. 8c is exactly what occurs after hearing the Tristan chord at the beginning of Wagner's Prelude. The dissonance of the apoggiatura in the third bar contrasts with the great harmonic relaxation produced by the chord itself (example 1),

Example 1. Tristan und Isolde (Prelude). R. Wagner

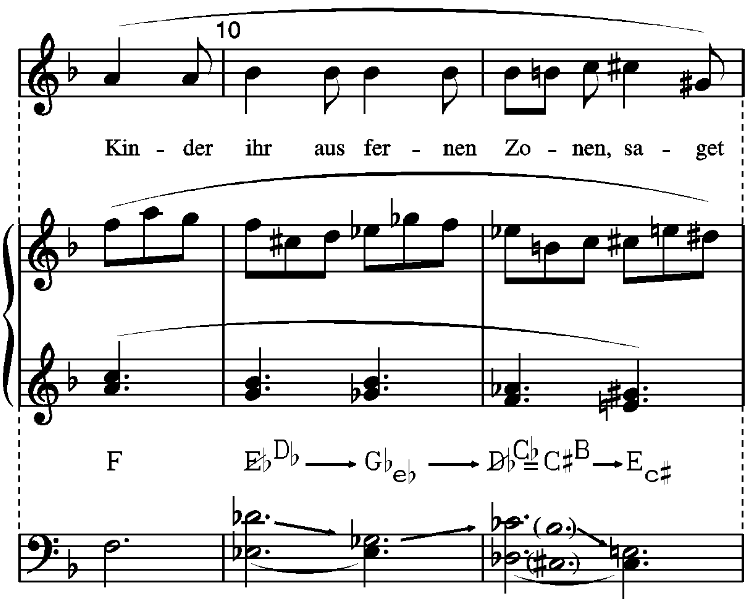
Examples 1, 2, 3 and 4 show different applications of the resolution between different chord fundamentals using the convergent chords theory. An arrow indicates local harmonic relaxation. The fundamentals are shown on the lower stave of each example: a minim stands for a functional fundamental (a fundamental including the major third, notated in capitals), and a crotchet stands for a non-functional fundamental (no major third and notated in small letters).

Example 2. Verklärte Nacht. A. Schoenberg



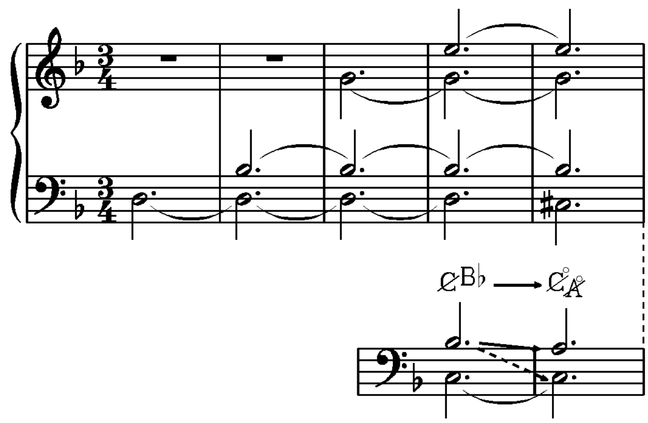
Harmonic resolutions are more easily explained by the symbols used in the examples than in words. We could summarise by saying that in example 2 (Verklärte Nacht) there is a clear harmonic resolution (phrigian) between the fundamentals of bar 42, although the final chord is not very consonant. This chord also resolves because it repeats fundamental F of the first chord of bar 41, and the other fundamental (G) is a fifth below fundamental

Example 3. Im Treibhaus (Wesendonk lieder). R. Wagner



(d) of that first chord. In example 3 (Im Treibhaus) there are cadential successions between chords, because one of the fundamentals always makes a jump of a fifth while the other continues into the following chord. Example 4 (Dormienti Ubriachi) is similar. Here second order (phrigian) resolution is used: one (virtual) fundamental is maintained

Example 4. Dormienti Ubriachi. A. Vivaldi



while the other moves a minor second down. There are many possible combinations between fundamentals similar to those shown in the examples, and this can be a help both for compositional purposes and to clarify cadential successions which cannot be explained clearly by the grades of classical tonality. It must be made clear that the successions we have studied are relaxations of local tension and are clearly influenced by the tonal field in which they are situated. A fundamental, functional or non-functional, will have added (tonal) resolution if it coincides with the tonic or the dominant of the strongest vector of the tonal field.

Separating chords according our approach to virtual pitch can also help to determine the tonal centres of works with weak tonal fields.

Development of classical tonality in the 17th. and 18th. centuries was acoustically based on both the melodic tendency that harmonic E shows towards pitch F (in order to resolve the "false fifths", as shown in fig. 6a) and on the fact that the harmonic structure CEBb is represented by pitch C, which in turn is generated by F (as 3rd.harmonic). Both ways there is a strong tendency towards F. Expressed in other words pitches C, E, and Bb create a tonal centre on F. If we play the single note of C or the chord of C major, disagreeing to treatises on harmony we are not in the key of C but in that of F.

This is nothing new. Since Riemann's time it has been known that tonic, subdominant and dominant chords (the latter with leading note) establish a tonal centre. Following the reasoning set out above, and transferring our language to that of the language of classical tonality the wording is briefer and differs slightly: considered as single notes, the dominant, leading tone and subdominant (notes C, E, and Bb in the key of F) determine a tonal centre —the tonic is taken for granted.

The practical method which is traditionally used by most musicians to find tonal centres could be defined as "following the leading tone". This method is not explicitly defined in musical theory, but it is the most practical way of ascertaining modulations at first sight. A slower but more reliable system is to use the functional theory of the grades of a diatonic scale. These methods work well enough in pieces whose key changes are not too complex, but in more chromatic works with weaker tonal fields, they are much more difficult to use. In these more chromatic pieces, depending on whether the composer is thinking "harmonically" or "melodically", there are often two ways of representing the same note, and the true leading tone may be hidden enharmonically.

Our system of fundamentals can shed new light on this. If notes C, E, and Bb determine a tonal centre, once we have separated the chords into convergent chords we only need to find fundamental C° or look for the interval of a major second between fundamentals. This is because if we find fundamentals Bb or bb and C, we know we have notes C, E and Bb. If the fundamentals are virtual the tonal effect is the same. With a little practice, finding the functional fundamentals of any chord involves no more than looking for the interval-classes of a major third or tritone (C-E, E-Bb).

Chords C° and CBb contain the notes C (real or virtual), E and Bb, and so these chords alone give a tonal centre on F (the C must be a functional fundamental, but not necessarily the Bb). The Tristan chord in example 1 for instance, is a chord of this type C#B, and therefore gives a local tonal centre on F#. The following bar gives a local tonal centre on A (E°). The overall tonal field perceived by the auditory system at any given moment is a combination of the foregoing tonal vectors and the degree of harmonic repose of each chord, depending on the local relaxations of tension mentioned earlier. The Tristan example begins in the key of F (taking into account the E at the end of the bar, although up to this point we have really been in Bb); we have a passing tonal centre on F# and a tonal centre of repose on A thanks to the local harmonic resolutions of a fifth and a minor second of the foregoing fundamentals in E°. I have given the name "homotonic resolutions" to these local harmonic resolutions (P5 and m2) which are independent of tonality resolutions.

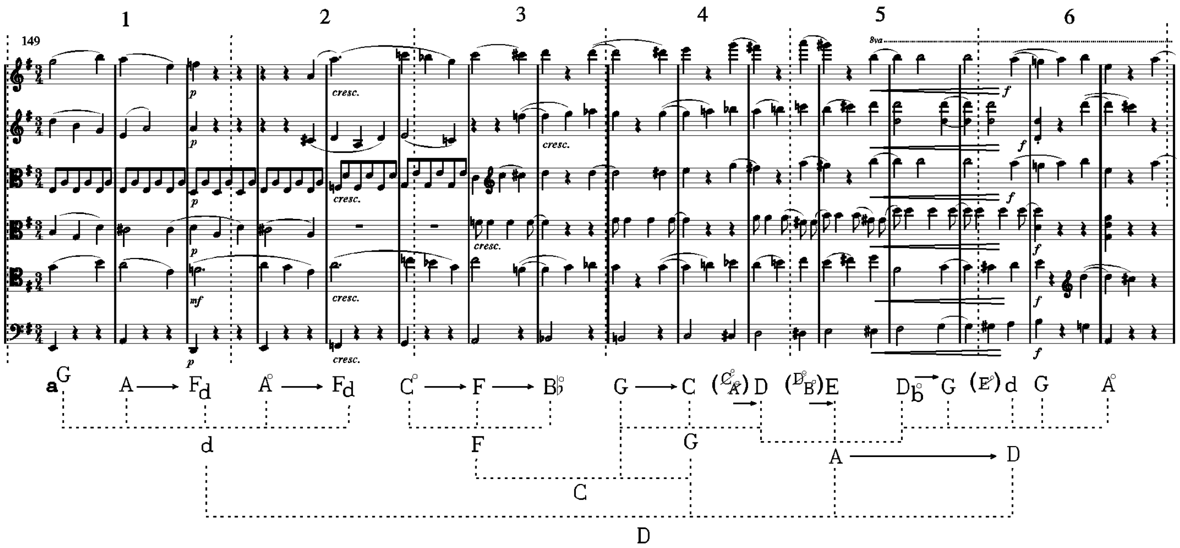
The presence of the interval of a major second between fundamentals also gives a tonal centre if the interval is found in different chords: it is equivalent, in functional harmony, to finding locally the functions of subdominant and dominant.

The clarity with which tonal centres are perceived is obviously also dependent on the tempo of the piece in question.

I will mention another case involving the position of tonal centres (example 5), not on account of any special difficulty, but due to the fact that it has been the subject of several studies.

Leman (1995a, 1995b) applied his "Model of Retroactive Tone-Center Perception" to bars 149-164 of Brahms' Second Sextet (fig.9). Leman's model is an acoustic one, i.e it is not applied to the printed score but to music heard live or on disc. Leman bases his system "on an auditory model that transforms musical signals into virtual pitch patterns" (called completion images). In his place-time model, the completion image is based on a periodicity analysis of neural firing pattern in auditory channels.

Example 5. Tonal centers in Brahms' Sextet num.2 (Allegro, bars 149-164)



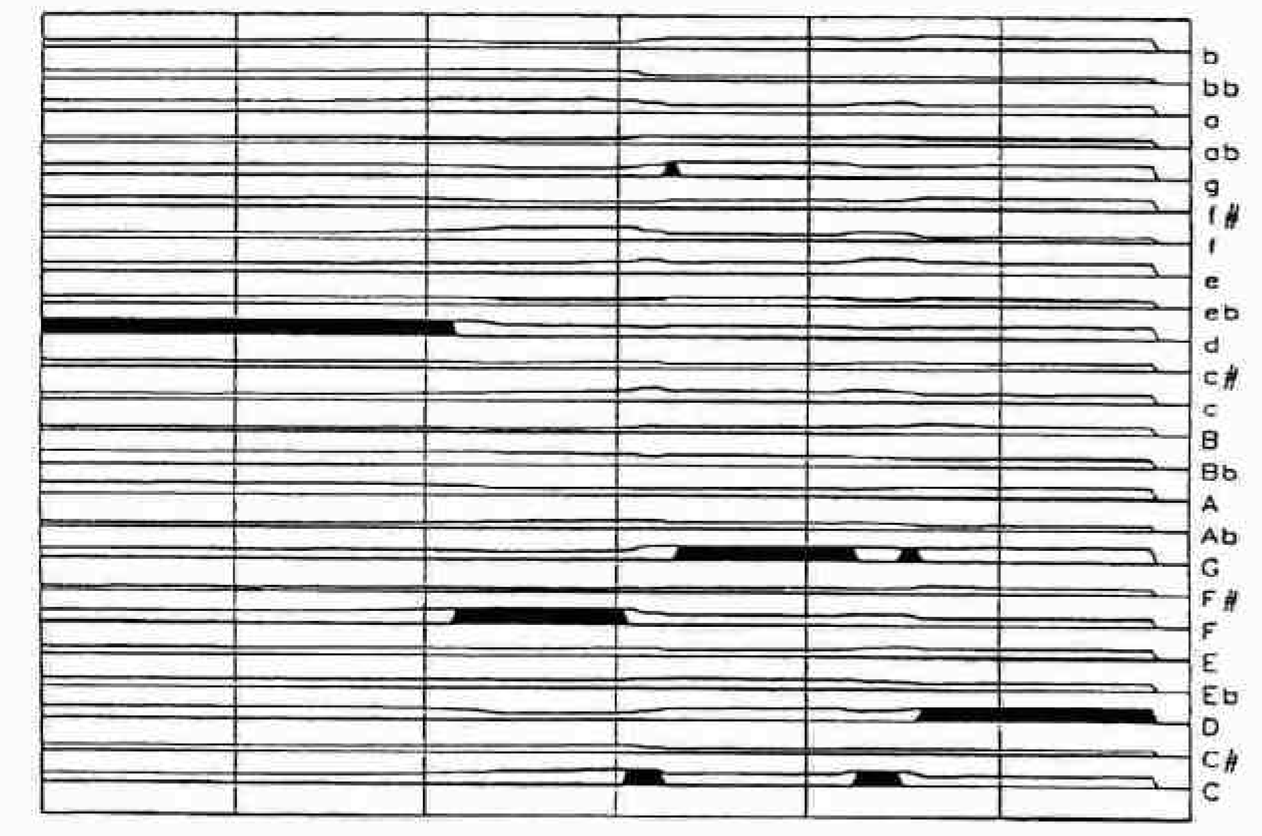


Fig. 9. Tonal centres in Brahms' Sextet num. 2 (Leman 1995a)

As can be seen by comparing example 5 and fig.8 both models give similar results. Both models take into account the basic principles of virtual pitch perception, our model being applied to the score and Leman's to live or recorded music.

The only difference between the two models occurs around bars 159 and 160 where I find a tonal centre on A while fig. 8 shows G and C.

While on the subject of Leman's model it must be mentioned that our model does not differentiate between modes when dealing with tonal centres. G major and minor have the same tonal centre —pitch class G— even though the colour of the music is different. Modal colour is caused mainly by the third note of the scale, B, which in G major (and other modes) causes tension towards C major/minor; this does not occur in the case of G minor (and other modes).

Summary and conclusion

In analysis of music both before and after classical tonality musical theory is often at odds to explain the cause of certain sensations of musical relaxation which occur at cadences and harmonic resolutions.

Resolution of harmonic tension can occur in three different ways —by (con)sonance, by relaxation of local harmonic tension and by tonal relaxation. In the course of this article we have worked out a model, based on certain properties of the auditory system, which I think will shed new light on the study of the last two types of resolution —relaxation of local harmonic tension (homotonic relaxation) and tonal relaxation. A practical advantage of this model is that it can be used on the printed score: unlike other theories the music does not have to be entered into a computer for analysis to be carried out.

This model is based on extracting the two main fundamentals which are to be found in 93.3% of chords of less than 6 notes. To do this I apply the basic concepts of virtual pitch but taking into account only those harmonics which are in a prime position in the first seven: I only consider harmonics 2, 3, 5 and 7, which in my opinion include the rest of the harmonics as far as the human auditory system is concerned.

With this model we discover new information concerning the tendencies of chords towards resolution. A fundamental (I use this term instead of "root") is "functional" when as well as representing a chord or a part thereof, it contains its major third. This gives internal tension to the part of the chord which represents this fundamental, a tension which can resolve locally in two main ways —dropping the fundamental a fifth or a minor second (or raising it by the complementary intervals, a fourth or a major seventh). This tension caused by the major third is deduced from the tension the fifth harmonic of any (harmonic) tone always has with the fundamental and the seventh harmonic. This tension is reflected in the two "false fifths" which are produced by these intervals and which surprisingly resolve in the same way.

Functional fundamentals, which can be either virtual fundamentals or form part of the chord, are the most informative when we are analysing tensions between successions of chords or looking for tonal centres. With complex chords the functional fundamentals can be found quickly by looking for the interval classes of a major third or a diminished fifth (as a functional fundamental can be a virtual one).

A chord is local harmonic (homotonic) relaxation of the foregoing chord if its fundamental(s) are also a relaxation after the foregoing fundamental(s). An interval of a major second between fundamentals determines a tonal centre.

A simple notation system for the functional properties of the fundamentals gives us a harmonic map of the score and a clarifying outline of the tonal and harmonic tensions which are in play.

FootNotes.

1. Table 1 in Terhardt, Stoll & Seewann (1982c)

2. Rameau says of "basse fondamentale" that "each sound of this fundamental bass represents a generator and is recognised at the same time as the immediate cause of all the musical effects" (*chaucun des sons de cette basse (fondamentale) représentant un générateur, se fait reconnoitre en même-temps pour la cause immédiate de tous les effets musicaux*). (Démonstration du principe de l'harmonie, 1750)

3. Tartini agrees with modern functional harmonic theory in considering Bb to be the virtual root ("Terzo suono") of the diminished fifth D:Ab. (De' principj dell'armonia musicale, 1767, p.85)

4. Inversions considered as a change of bass, not as interval symmetry.

5. The reasoning is as follows:
Following Parncutt's model I will use the following terms for the intervals —P1 (unison, octave), M2 (major second), M3 (major third), m3 (minor third), P5 (perfect fifth), and so on.
Parncutt maintains that an interval of a minor third also gives indirect support to the bass. He argues that the 3rd. and 5th. harmonics, of a fundamental —P5 and M3— can also be taken as the 7th. and 3rd. harmonics of a theoretical pitch a minor third below this fundamental. For example, tone C has G as its 3rd. harmonic (P5) and E as its 5th. harmonic (M3). But G can also be taken as the 7th. harmonic (m7) of tone A, and E as the 3rd. harmonic of the same tone. That is the auditory system understand a kind of subharmonic or virtual pitch (?) a minor third below the fundamental.
My opinion is that when Parncutt includes interval m3 as support for the bass, he is detracting from the theory of virtual pitch. His theory is tantamount to saying that the auditory system somehow recognises a subharmonic a minor third below the root, a daring assertion. A can be taken as a subharmonic of G and E; in the same way it could be said that Eb can be a subharmonic of G and Bb, which are also both harmonics of C, thereby adding an interval of a major sixth as support for a root. This is in contradiction with the minor third as support for a root.

6. The "senario" can in fact be reduced to the relations between the numbers 1, 2, 3 and 5, as 4 and 6 are multiples of 2 and 3. The Pythagorean School only took into account the first three numbers, 1, 2 and 3.

7.
7/4= 1,75
4/3 x 4/3= 1,777...
In fact, in daily musical practice, these numerical interval relations never occur. However, studies such as those by Fransson, Sundberg & Tjernlund (1974) seem to show that there can be considerable variations in the tuning of a performance without loss of the notion of tonal and harmonic structure which is determined by these arithmetic relations. An example is the artificial equal-temperament tunning.

8. Harmonics above no. 24 are completely imperceptible due to the masking effect which occurs.

9. I do not know whether this experiment was carried out in an objective manner. Have experimented myself and it seems to work quite well with pure tuning.

10. Leverie S. (1992). *International Journal of Musicology, Vol. 1*

11. In traditional tonality, with use of the resolution of fundamentals C°-B, the Bb is usually harmonised as an A# —chord of an augmented sixth— and the fifths so produced are known as Mozartian fifths.

12.Always with reference to relaxations of local tension. The plagal cadence F-C is produced by a strong tonal memory and the relaxation afforded by coming to rest on the tonic chord. Locally, however, it is a succession of tension.
Besides these two harmonic resolutions between fundamentals, our model has two other weaker harmonic resolutions. These are not shown in this article, but they involve the fundamental rising a major second (or falling a minor seventh) and rising a major third (or falling a minor sixth).

References

Balsach, Ll. (1994). *La convergència harmònica. Morfogènesi dels acords i de les escales musicals*, Barcelona. Clivis Publications.
Forte, A. (1973). The Structure of Atonal Music, New Haven & London. Yale University Press
Helmholtz, H. (1863). *Die Lehre von den Tonempfindungen als physiologie Grundlage für Theorie der Musik*, Brumswick: Longmans&Co (Eng. ver..: On the Sensations of Tone, 1875).
Hindemith, P. (1937). *Unterweisung im Tonsatz*, Mainz: Schott & Co. (Eng. ver.: *The craft of musical composition*, London&New York 1942).
Krumhansl, C-L. (1990). *Cognitive Foundations of Musical Pitch*, New York & Oxford: Oxfod University Press
Leman, M. (1995a). A model of Retroactive Tone-Center Perception. *Music Perception*, Vol.12(4), 439-471.
Leman, M. (1995b). *Music and Schema Theory. Cognitive Foundations of Systematic Musicology*, Berlin-Heidelberg: Springer Verlag
Levarie, S. (1992). Musical Polarity: Major and Minor. *International Journal of Musicology* 1, 29-45.
Luppi, A. (1989). *Lo Specchio dell'Armonia universale*, Milano: Franco Angeli Libri
Meddis, R. & Hewitt, J. (1991). Virtual pitch and phase sensitivity of a computer model of the auditory periphery. I: Pitch identification. *Journal of the Acoustical Society of America*, 89(6), 2866-2882.
Parncutt, R. (1988). Revision of Terhardt's Psychoacoustical Model of the Root(s) of a Musical Chord. *Music Perception*, Vol.6(1), 65-93.
Plomp, R. (1964). The ear as a Frequency Analyzer. *Journal of the Acoustical Society of America*, 36, 1628-1636.
Rameau, J-P. (1722). *Traité de l'Harmonie Réduite à ses Principes Naturels*, Paris: Jean Baptiste-Christiphe

Rameau, J-P. (1750). *Démonstration du Principe de l'Harmonie*, Paris: Ballard
Riemann, H. (1893). *Vereinfachte Harmonielehre oder die Lehre von den tonalen Funktionen der Akkorde*, London & New York (Editorial Labor: spanish translation) (Eng. ver.: 1896)
Tartini, G. (1754). *Trattato di musica secondo la vera scienza dell'armonia*, Padua: Stampa del seminario facsimil: Casa Editrice Dott. Antonio Milani.
Tartini, G. (1767). *De' principj dell'armonia musicale*, Padua: Stampa del seminario facsimil: Casa Editrice Dott. Antonio Milani.
Terhardt, E. (1974). Pitch, consonance, and harmony. *Journal of the Acoustical Society of America*, 55(5), 1061-1069.
Terhardt, E. (1982a). Die psychoakustischen Grundlagen der musicalischen Akkordgrundtöne und deren algorithmische Bestimmung. In C.Dahlhaus & M.Krause (Eds.). *Tiefenstruktur der Musik*. Berlin: Technical University of Berlin.
Terhardt, E., Stoll, G. & Seewann, M. (1982b). Algorithm for extraction of pitch and pitch salience from complex tonal signals. *Journal of the Acoustical Society of America*, 71(3), 679-688.
Terhardt, E., Stoll, G. & Seewann, M. (1982c). Pitch of complex signals according to virtual-pitch theory: Tests, examples, and predictions. *Journal of the Acoustical Society of America*, 71(3), 671-678.
Van Immerseel, L. & Martens, J-P. (1992). Pitch and voiced/unvoiced determination with an auditory model. *Journal of the Acoustical Society of America*, 91(6), 3511-3526.
Vogel, M. (1975). *Die Lehre von den Tonbeziehungen*, Bonn: Orpheus (Verag für systematische Musikwissenschaft) (Eng. transl.: *On the Relations of Tone*, Bonn 1993).